THE SCIENCE ASPECTS OF PHOSPHORUS TRADING

Christopher G. Uchrin
Simple TP Mass-Balance Model
Schematic
Simple TP Mass-Balance Model

Model Equation

\[ Q \ P_i - Q \ P - k_s \ P \ V = V \ \frac{dP}{dt} \]

\[ k_s = \text{sedimentation coefficient} = \alpha \ \frac{v_s}{H} \]

\[ \alpha = \frac{\text{particulate P}}{\text{TP}} \]

\[ v_s = \text{sedimentation velocity} \]

\[ H = \text{average depth} \]
Simple TP Mass-Balance Model
Steady-State Solution

\[ P = P_i \left[ \frac{1}{1 + k_s \tau} \right] \]

\( \tau = \) hydraulic detention time
\[ = \frac{V}{Q} \]
Simple TP Mass-Balance Model
Trapping Efficiency

$$R = 1 - \left( \frac{P}{P_i} \right)$$
CASE STUDY

Carnegie Lake Watershed

(Jenq, Granstrom, Hsueh, and Uchrin, 1984)
Peter Bk.
Woodsville Bk.

Stony Bk Br.
Baldwin Ck.
Lewis Bk.

Honey Br.
Princeton Farm STP

Cleveland Bk.
Western E1. STP
PSM, Prety Bk STP

MP 0.5 Princeton Theo STP
MP 0.0

STONY BROOK SCHEMATIC
IN-STREAM PHOSPHORUS MODEL EQUATION:

\[ \frac{\partial P}{\partial t} = \frac{Q}{A} \frac{\partial P}{\partial x} - KP \]
MILLSTONE RIVER PHOSPHORUS CALIBRATION - LOW FLOW
OBJECTIVE FUNCTION

\[ \sum_{k=1}^{M} \sum_{i=1}^{N} \left[ C_{ki} (X_{ki}) + C_{ki} (Y_{ki}) + C_{ki} (Z_{ki}) \right] \]

i = tributary counter
j = reach counter
CONSTRAINTS

\[ P_{ki} \leq S_{ki} \]

- \( S_{ki} = 0.10 \text{ mg/L instream} \)
- \( S_{ki} = 0.05 \text{ mg/L point of lake input} \)
Cost Function (1980 $s) for TP Removal
WWTP Sources

Figure 4. Annual Cost – C(X) [x1000/MGD] vs. Percent TP Removal – X for Point Sources.
Cost Function (1980 $s) for TP Removal
Urban and Rural Stormwater Sources

Figure 5. Annual Costs - C(Y) ($/hectare) vs. Percent TP Removal - Y for Urban Nonpoint Sources and C(Z) ($/hectare) vs. Percent TP Removal - Z for Rural Nonpoint Sources.